Analysis of Various Bets in Yo!

Introduction

This paper will discuss the house advantage of various bets in the proposed casino game Yo! This is neither final nor exhaustive, but rather a specific analysis of the proposed payouts for several wagers. This analysis will not go into great depth regarding the overall gameplay or other procedural matters unless that information is pertinent to the analysis, and assumes the reader has a basic knowledge of how the game is played. The following are the bets to be analyzed:

1. **PLAY** - the PLAY wager is analogous to the PASS LINE bet in ordinary craps. The house advantage will be calculated for a few variations of this bet.
2. **MORE** - the MORE wager is analogous to the ODDS bet in ordinary craps.
3. **YO! (bonus wager)** - the YO! wager is a bonus bet that is one of the many ways the game Yo! is different from the game of craps. This is the only bet in Yo! that incorporates the second pair of dice.
4. **Various one roll wagers** - the wagers in 1, 2, and 3 above are not necessarily settled on each individual roll, but several wagers in the game of Yo! are. These wagers fall into the category of one roll wagers, and will be analyzed in this section. These bets include:
   - Pairs
   - Any 2, 3, or 12
   - Eleven
   - Low 4, 5, or 6
   - High 8, 9, or 10

If you're impatient, skip to the last page to see a summary of the house advantages for each of these bets. If you'd like more detail, read the pages that follow.

1. The PLAY Wager

The PLAY wager can be settled in one roll or in multiple rolls, depending on the roll sequence. After the PLAY wager is placed, the two game dice[1] are rolled. If an 11 is rolled[2] on the game dice, this bet is an automatic winner. If a 2, 3, or a 12 is rolled, this bet is an automatic loser. If a 4, 5, 6, 8, 9, or 10 is rolled, the dice are rolled repeatedly until either that number is rolled again, which will result in a winner, or a 7 is rolled, which will result in a loser.

I did not indicate what happens when a 7 is rolled on the initial roll–this is because we will analyze several scenarios with different outcomes for the 7 (i.e. the 7 wins on the initial roll, or the 7 loses on the initial roll, etc.)

Ordinary craps

In ordinary craps, the 7 and the 11 are automatic even money (1-to-1) winners on the initial roll. The 2, 3, and 12 are automatic losers and the remaining sum of dice values must be rolled again prior to a 7 rolling in order to win even money. See Table 1 below for a summary of these payouts.

Table 1: Initial roll outcomes and payouts in ordinary craps

Sum of game dice	Outcome	Payout
7 or 11	win	1-to-1
2, 3, or 12	lose	-1-to-1
4, 5, 6, 8, 9, 10	roll again	-

In the case where you must roll again to determine the outcome of the wager, rolls will continue until either the number rolled on the initial roll is rolled again (a win, payout = 1-to-1) or a 7 is rolled (a loss, -1-to-1).

There are a finite number of ways to roll the dice (each one only has six faces, so there are $6^2 = 36$ ways to arrange two six sided dice). We can count the number of ways each sum of the two faces of the two dice can be made. For example, there is only one combination that results in a sum of 2–if both dice are showing one pip. Similarly, there are 6 ways to roll a 7. Let (a, b) be the result of a roll of two six sided dice where a is the result of one distinct die and b is the result of the other. The set equal to all possible outcomes that result in a sum of 7 is $\{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}$. See Table 2 for the number of ways each sum can be made.

[1] For the sake of this analysis, I will call the two main numeric dice in this game the "game dice" and the two numeric dice that only come into play for the YO! bonus wager the"bonus dice."

[2] A number is considered to be"rolled" when the sum of the two dice in question is equal to that number.

Table 2: Number of ways to roll each sum of two six sided dice

Sum of dice	Number of ways
2	1
3	2
4	3
5	4
6	5
7	6
8	5
9	4
10	3
11	2
12	1
Total	36

Using this information, we can determine the house advantage of the PLAY bet under ordinary craps payouts. See Table 3 for the initial roll outcomes and payouts in Table 1 with the number of ways and probability of rolling added.

Table 3: Initial roll outcomes and payouts in ordinary craps with probabilities

Sum of game dice	Outcome	Payout	Number of ways	Probability
7 or 11	win	1-to-1	8	$\frac{8}{36} = .222222$
2, 3, or 12	lose	-1-to-1	4	$\frac{4}{36} = .111111$
4, 5, 6, 8, 9, 10	roll again	-	24	$\frac{24}{36} = .666667$

If there was a payout in each row, we could easily calculate the house advantage of the PLAY wager since we have probabilities for each outcome. Unfortunately, we need to do a bit more work since rolling the 4, 5, 6, 8, 9, or 10 on the initial roll does not immediately result in a win or a loss. These sums account for 24 out of 36 of the possible ways to initially roll the dice. This means that the game will have a decision after the initial roll one third of the time and require multiple rolls for a decision two thirds of the time. We know the payouts and probabilities when an immediate decision is made; we will now determine the payout when multiple rolls are required.

Consider Table 4, which shows the win probabilities for each of the possible multiple roll decisions.

Table 4: Win probabilities for decisions requiring multiple rolls

Sum of game dice	Number of ways to win	Number of ways to lose	Probability of winning	Probability of losing
4	3	6	.333333	.666667
5	4	6	.4	.6
6	5	6	.454545	.545455
8	5	6	.454545	.545455
9	4	6	.4	.6
10	3	6	.333333	.666667

Now, see Table 5 below, which shows the calculation of the value of $1 when the game requires multiple rolls.

Table 5: Value of $1 when game requires multiple rolls

Point[3] ($= x$)	$P(x$ is the point $\mid$ there is a point)	P(win)	P(lose)	P(win $\mid$ x is the point)	P(lose $\mid$ x is the point)	E[$1 $\mid$ x is the point]
4	.125	.333333	.666667	.041667	.083333	-.041667
5	.166667	.4	.6	.066667	.1	-.033333
6	.208333	.454545	.545455	.094697	.113636	-.018939
8	.208333	.454545	.545455	.094697	.113636	-.018939
9	.166667	.4	.6	.066667	.1	-.033333
10	.125	.333333	.666667	.041667	.083333	-.041667
Total						-.187879

This means that when the game goes to multiple rolls (which happens two thirds of the time, as seen in Table 3), the expected value of one dollar wagered is equal to $-\$0.187879$. In other words, the average payoff when a point is established is -18.7879 cents for every dollar bet. Now we can calculate the overall value of $1 bet (and subsequently the house advantage) for the PLAY bet under ordinary craps rules. See Table 6 for this development.

Table 6: House advantage of PLAY wager, ordinary craps rules

Initial roll	Probability	Average payoff	E[$1]
7 or 11	.222222	+1	.222222
2, 3, or 12	.111111	-1	-.111111
4, 5, 6, 8, 9, or 10	.666667	-.187879	-.125253
6 Total			-.014142
House advantage (%)			1.4142%

[3] I am going to call the sum of the two dice (which must be rolled before a 7 in order for the PLAY wager to be a winner) the "point" both for simplicity and to be consistent with ordinary craps terminology.

In other words, for each dollar wagered on the PLAY wager, under ordinary craps rules, the player will lose 1.4142 cents on average and the house advantage is 1.4142%. This matches the published house advantage of the Pass Line wager in craps, widely available from several sources and considered public knowledge. Now we'll examine some variations to the ordinary craps pay table.

7 pushes, 11 pays double

Under this proposed scenario, the seven would neither win nor lose on the initial roll and the eleven would pay 2-to-1 instead of the customary 1-to-1. These are the only differences from ordinary craps. We would expect the overall house advantage under this pay table to be greater than that of ordinary craps since we are taking away 6 winners and only doubling the payout for 2 winners.

Note that the expectation does not change for the case when multiple rolls are required (i.e. the first roll is a 4, 5, 6, 8, 9, or 10). Since we are counting the seven as a push, however, we can eliminate those six rolls as having a decision (there is no impactful outcome from a 7 on the initial roll since it neither wins nor loses–it might as well have not happened). We are calculating the house advantage per decision, not per individual roll, so we can eliminate these six initial roll outcomes from the house advantage analysis. This leaves only 30 rolls that matter (the ordinary 36 less those that result in 7). This changes the probability that a decision will require multiple rolls (from $\frac{24}{36}$ in the "ordinary" case to $\frac{24}{30}$ in this case).

We can recreate a table just like Table 6 previously, using numbers that we've already determined. See Table 7 below for a development of the house advantage under this pay table.

Table 7: House advantage of PLAY wager, 7 pushes, 11 pays double

Initial roll[4]	Probability	Average payoff	$E[\$1]$
11	.066667	+2	.133333
2, 3, or 12	.133333	-1	-.133333
4, 5, 6, 8, 9, or 10	.8	-.187879	-.150303
Total			-.150303
House advantage (%)			15.0303%

As you can see, the house advantage is significantly higher than the ordinary craps scenario. If the intent of this bet is to keep it the bet with the lowest non-zero edge on the table, using this pay table is definitely not the way to go.

[4]You may notice that the 7 is missing here and wonder why it is omitted. Certainly the 7 could be the outcome of the roll (and will be one out of every six rolls, on average), but what we are examining here are the rolls that have an impact on the game and these probabilities are conditional probabilities given that the roll is impactful.

A house advantage this high, in conjunction with those on the one roll bets, would be a deterrent for any savvy players. Further, even non-savvy players would likely soon find the game unplayable due to the overall edge.

7 pushes, 11 pays quadruple

This scenario is very similar to the previous one, with the only difference being that the 11 would pay 4-to-1 rather than 2-to-1. This is an interesting example because it brings the house edge almost all the way down to the same edge as "ordinary" craps, which has proven to be a sustainable edge from both the players' and the houses' perspectives. See Table 8 for the development of the house advantage under this scenario.

Table 8: House advantage of PLAY wager, 7 pushes, 11 pays double

Initial roll	Probability	Average payoff	$E[\$1]$
11	.066667	+4	.266667
2, 3, or 12	.133333	-1	-.133333
4, 5, 6, 8, 9, or 10	.8	-.187879	-.150303
Total			-.016970
House advantage (%)			1.6970%

As mentioned, this edge is very similar to the edge under the ordinary craps rules where the 7 and 11 both win and are paid 1-to-1. Yo! is eleven-centric, so this pay table could be an attractive option as the 11 pays four times the usual amount–at the expense of the 7 paying nothing (but not losing, either).

Various other pay tables can be examined in a similar fashion. Since final pay tables have not been determined, the house edges quoted here are just for illustrative purposes. Final house edges can be published when official pay tables are determined.

2. The MORE Wager

The MORE wager is exactly analogous to the Odds wager in craps. Players can place up to a certain amount, usually a multiple of their PLAY (Pass Line in craps) bet, in the MORE spot after a point has been established.

This MORE bet wins when the PLAY bet wins and loses when the PLAY bet loses, but it pays better than the PLAY wager (i.e. more than 1-to-1). In fact, the MORE wager pays in exact proportion to the likelihood of winning the bet, and as such, there is no house advantage on this wager.

For example, if the 4 is the point, then you're twice as likely to roll a 7 and lose than you are to throw a 4 and win, so the MORE wager pays you 2-to-1. We could go through an expected value calculation, but it is intuitive that the advantage on this wager is zero. For the sake of simplicity and time, we won't further analyze this wager since there is no advantage to either the house or the player and thus it has zero impact on the house's expected take or the player's expected winnings.

I will add that there is a small difference in this bet as compared to the Odds wager in craps–once players place the MORE bet, it must remain on the table in Yo! whereas the players can remove, reduce, or increase their odds wager at any time in craps. Alas, this has zero effect on the advantage.

3. The YO! (bonus) Wager

The YO! bonus wager is a bet that incorporates the secondary or bonus dice in Yo! In fact, it is the only wager that depends on these dice.

This bet has been discussed ad nauseam in previous analyses (which I can re-provide if requested), so the discussion here will be abbreviated.

If the bonus dice, which are two six sided ordinary dice but in a different color than the game dice (so that they may be differentiated from the game dice), are rolled such that the sum of the two faces is 11, the YO! bonus wager is a winner. If the outcome of rolling the bonus dice is a sum other than 11, the bonus neither wins nor loses. The YO! bonus wager only loses when a 7 is thrown on the game dice (and 11 is NOT thrown on the bonus dice). In the unique scenario when the result of the game dice is a 7 but the result of the bonus dice is 11, the YO! bonus bet is a winner.

The payout when the YO! bonus bet is a winner is determined based on the value of the game dice. We'll analyze the proposed table and also provide a general formula for determining the house edge, but I'll leave the majority of the exercise of deriving that formula up to the reader. The proposed pay table for the YO! bonus wager can be found in Table 9.

Table 9: Proposed pay table for the YO! bonus wager

11 on bonus dice with (on game dice)	Payout
11	20-to-1
Any pair	4-to-1
Any other sum	1-to-1

In order to determine the edge on this wager under the suggested pay table, let's look at the total number of ways to roll all four (two bonus and two game) dice. See Table 10.

Table 10: Possible dice and YO! bonus bet outcomes

Sum of game dice	11 on bonus dice	Outcome for YO! bonus bet
2	YES	Win
3	YES	Win
4	YES	Win
5	YES	Win
6	YES	Win
7	YES	Win
8	YES	Win
9	YES	Win
10	YES	Win
11	YES	Win
12	YES	Win
2	NO	Push
3	NO	Push
4	NO	Push
5	NO	Push
6	NO	Push
7	NO	Loss
8	NO	Push
9	NO	Push
10	NO	Push
11	NO	Push
12	NO	Push

In Table 11, let's add the number of ways to roll each of these scenarios. This table will summarize all 1,296 ways to roll the four dice ($= 6^4$).

Table 11: Possible dice and YO! bonus bet outcomes with number of ways

Sum of game dice	11 on bonus dice?	Outcome for YO! bonus bet	Number of ways
2	YES	Win	2
3	YES	Win	4
4	YES	Win	6
5	YES	Win	8
6	YES	Win	10
7	YES	Win	12
8	YES	Win	10
9	YES	Win	8
10	YES	Win	6
11	YES	Win	4
12	YES	Win	2
ANY Except 7	NO	Push	1020
7	NO	Loss	204
Total			1296

The total number of ways to roll the dice found in the last row is 1,296, which is the number we expected. Only 276 of these rolls actually affect the Yo! bonus bet. Specifically, there are 72 rolls that include an 11 on the bonus dice and 204 instances of a 7 on the game dice without an 11 on the bonus dice. On any of the other 1,020 rolls (the ones without an 11 on the bonus dice or a 7 on the game dice), the YO! bonus bet neither wins nor loses.

The main result here is that there are 72 ways to win the YO! bonus bet and 204 ways to lose it. The majority of rolls (1020/1296 or 78.70%) will not affect the YO! bonus bet.

At this point, I'll offer the general formula for computing the house advantage on the Yo! bonus bet, and then we will verify the formula for the case of the proposed pay table. Let Table 12 summarize the general pay table for the YO! bonus bet.

Table 12: General pay table for the YO! bonus bet

Outcome of game dice[5]	Number of ways	Payout
2	2	a
3	4	b
4 (pair[6])	2	c
4 (non pair)	4	d
5	8	e
6 (pair)	2	f
6 (non pair)	8	g
7	12	h
8 (pair)	2	i
8 (non pair)	8	j
9	8	k
10 (pair)	2	l
10 (non pair)	4	m
11	4	n
12	2	o

Now, let

$$\Sigma = a + 2b + c + 2d + 4e + f + 4g + 6h + i + 4j + 4k + l + 2m + 2n + o$$

Then, the house edge (HE) is

$$HE = \frac{102 - \Sigma}{138}$$

In the case of the proposed YO! bonus bet pay table, $n = 20$, $a = c = f = i = l = o = 4$, and $b = d = e = g = h = j = k = m = 1$. So,

$$\Sigma = 4 + 2 + 4 + 2 + 4 + 4 + 4 + 6 + 4 + 4 + 4 + 4 + 2 + 40 + 4 = 92$$

and

$$HE = \frac{102 - 92}{138} = \frac{10}{138} = 7.2464\%$$

We can verify this value by calculating the house edge more directly. Let's add some columns to the proposed pay table, see Table 13.

[6] "pair" means that the 4 is rolled as a 2 on one die and a 2 on the other

Table 13: Proposed pay table for the YO! bonus wager with average payoff

11 on bonus dice with (on the game dice)	Payout Payout	Number of ways	Probability	Probability * Payoff
11	20-to-1	4	$\frac{4}{72}$	$\frac{80}{72}$
Any pair	4-to-1	12	$\frac{12}{72}$	$\frac{48}{72}$
Any other sum	1-to-1	56	$\frac{56}{72}$	$\frac{56}{72}$
Total				$\frac{184}{72}$

This table shows that the average payoff for the YO! bonus bet according to this pay table is $\frac{184}{72}$. Recall that the YO! bonus bet wins $\frac{72}{276}$ of the time and loses $\frac{204}{276}$ of the time. Thus to compute the house edge (HE), we simply need to set up the following expected value equation

(average payoff)$*$(probability of winning)$-$(average loss)$*$(probability of losing)

We know each of these inputs. The average payoff in this case is $\frac{184}{72}$. The probability of winning is $\frac{72}{276}$. The probability of losing is $\frac{204}{276}$. We will use \$1 as the average loss amount. This can also be interpreted as "1 unit". The equation becomes

$$\left(\frac{184}{72} * \frac{72}{276}\right) - \left(1 * \frac{204}{276}\right)$$
$$= \frac{184}{276} - \frac{204}{276}$$
$$= -\frac{20}{276} = -.072464 = -7.2464\%$$

This equation was set up from the perspective of the player (win - loss), so the negative result indicates an advantage for the house. Just as we expected, the edge is 7.2464%.

4. Various one roll wagers

There are many wagers in Yo! that either win or lose with every roll of the dice. The game dice are the only two dice that affect these bets; the bonus dice are irrelevant.

Determining the house advantage of these bets is a simple since a decision is made with each roll of the dice. We determine the finite number of ways to win the bet and the finite number of ways to lose the bet. From this, we determine a win probability and a loss probability. Once we have these, coupled with the payouts, expected value and house advantage calculations are trivial. See Table 14 below for a summary of these wagers and their house advantages.

Table 14: House advantage of one roll wagers

Wager	Ways to win	Ways to lose	$P(\text{win})$	$P(\text{lose})$	Payoff	$E[\$1]$	HE
Pairs	4	32	.111111	.888889	7-to-1	-.111111	11.1111%
2, 3, or 12	4	32	.111111	.888889	7-to-1	-.111111	11.1111%
Eleven	2	34	.055556	.944444	15-to-1	-.111111	11.1111%
4, 5, or 6	12	24	.333333	.666667	3-to-2	-.166667	16.6667%
8, 9, or 10	12	24	.333333	.666667	3-to-2	-.166667	16.6667%

Conclusion

This document outlines the house advantage for several proposed bets in the dice game Yo! This is a working draft and is no means final. After final pay tables are determined, a formal document, complete with full explanations and calculations of house advantages and including all gameplay rules and table layouts will be provided.

For a summary of the house advantages on all the bets as currently proposed at the time of this writing (June 2, 2017), see Table 15 below.

Table 15: Summary of house advantages for proposed wagers

Wager	Payout	House advantage
PLAY - ordinary craps	1-to-1	1.4142%
PLAY - 7 push, 11 double	varies	15.303%
PLAY - 7 push, 11 quadruple	varies	1.6970%
MORE	varies	0.00%
YO! Bonus	varies	7.2464%
Pairs	7-to-1	11.1111%
Any 2, 3, or 12	7-to-1	11.1111%
Eleven	15-to-1	11.1111%
Low 4, 5, or 6	3-to-2	16.6667%
High 8, 9, or 10	3-to-2	16.6667%